AJAE Appendix:
'The Welfare Impacts of Commodity Price Volatility: Evidence from Rural Ethiopia'

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A. Agricultural Household Model

The derivations in this appendix closely follow those in Barrett (1996), who builds on Turnovsky et al.’s (1980) work on individual consumers and Finkelshtain and Chalfant’s (1991) work on price risk in the context of the agricultural household model. In what follows, we report the basics of the model. Readers interested in more detailed explanations and derivations of these findings are encouraged to consult those prior works.

Consider a representative agricultural household whose preferences are represented by a von Neumann-Morgenstern utility function $U(\cdot)$ defined over consumption of a vector $c_o = (c_{o1}, c_{o2}, ..., c_{oK})$ of K goods whose consumption and/or production is observed and whose associated stochastic price vector is $p_o = (p_{o1}, p_{o2}, ..., p_{oK})$; a composite c_u of all goods whose consumption and/or production is unobserved by the econometrician and whose associated stochastic composite price is p_u;[1] and leisure ℓ. The function $U(\cdot)$ is concave in each of its arguments, with the Inada condition $\frac{\partial U}{\partial x}\Big|_{x=0} = \infty$ with respect to each argument *x*.

[1] In order simplify the exposition, we refer to the vector of commodities whose consumption and production is unobserved by the econometrician as “the unobserved good” in what follows.

All K goods observed and the unobserved good can, in principle, be produced and consumed by the household.[2] The household has an endowment W^L of time and an endowment W^H of land. The production of each of the K observed commodities is denoted by

$$F_{oi}(L_{oi}, H_{oi}),\ i \in \{1, ..., K\}, \tag{A1}$$

where L_{oi} denotes the amount of labor used in producing observed commodity i and H_{oi} denotes the amount of cultivable land used in producing observed commodity i. The production of the unobserved good is denoted by

$$F_u(L_u, H_u), \tag{A2}$$

where L_u and H_u denote the amount of labor and cultivable land, respectively, used in producing the unobserved commodity. Both F_{oi} and F_u are strictly increasing but weakly concave in each argument.

Agricultural labor is a function of household labor on the farm L^f and of hired labor L^h, but note that those are imperfect substitutes given that monitoring of hired workers

[2] For example, it is quite common in developing countries for rural household to grow a staple crop (e.g., barley, wheat, maize, etc.) and many other non-staple crops (e.g., coffee, beans, etc.) For a specific crop, it is also common for some households to be net buyers of it, for some households to be autarkic with respect to it, and for some households to be net sellers of it. Finally, households may switch from one category – net buyer, autarkic, or net seller – to another from one period to the next (Bellemare and Barrett, 2006).

may be imperfect, with the usual moral hazard consequence. A general function $h(\cdot)$ maps hired labor into family labor equivalent units. The household can also sell a quantity L^m of labor on the market at parametric wage rate *w*, but the market for credit is assumed missing.

The household’s time constraint is such that $L^m + \ell + \sum_i L^f_{oi} + L^f_u \leq W^L$, where ℓ is the household’s leisure time; L^f_{oi} is the amount of household labor devoted to production of observed commodity *i* and L^f_u is the amount of household labor devoted to production of the unobserved good. The household’s land constraint is such that $H^m + H^f \leq W^H$, where H^m is the amount of household land leased out on the tenancy market at parametric rental rate r; and $H^f \equiv \sum_i H^f_{oi} + H^f_u$ is the amount of household land devoted to the production of the observable and unobservable commodities, respectively. Likewise, H^h_{oi} and H^h_u are the amounts of leased in land devoted to the production of the observable and unobservable commodities, respectively, so that $H_{oi} \equiv H^f_{oi} + H^h_{oi}$ and $H_u \equiv H^f_u + H^h_u$ are the total amounts of land allocated to the production of the observable and unobservable commodities. Finally, let *I* denote the household’s unearned income, i.e., income from transfers or remittances.

In what follows, we consider a two-period model. That is, all input prices are known and (stochastic) crop prices are unknown when production decisions are made in *t*, but post-harvest crop prices are revealed before consumption decisions are made in t + 1.

The household's problem is thus to

$$\max_{\{H_{oit}^h, H_{ut}^h, H_{ut}^f, L_t^m, L_{oit}^h, H_{oit}^f, L_{oit}^f, L_{ut}^h, L_{ut}^f, H_t^m, \ell_t\}} E \max_{\{c_{ot+1}, c_{ut+1}\}} U(c_{ot+1}, c_{ut+1}, \ell_t) \tag{A3}$$

subject to

$$p_{ot+1}c_{ot+1} + p_{ut}c_{ut+1} \leq EY^*, \tag{A4}$$

$$EY^* \equiv w_t[L_t^m - \sum_{oi} L_{oit}^h - L_{ut}^h] + r_t[H_t^m - \sum_{oit} H_{oit}^h - H_{ut}^h]$$

$$+ \sum_i p_{oit+1}F_{oit}(L_{oit}, H_{oit}) + p_{ut+1}F_{ut}(L_{ut}, H_{ut}) + I_t, \tag{A5}$$

$$L_{oit} \equiv h(L_{oit}^h) + L_{oit}^f \quad \forall i, \tag{A6}$$

$$L_{ut} \equiv h(L_{ut}^h) + L_{ut}^f, \tag{A7}$$

$$L_t^m + \ell_t + \sum_i L_{oit}^f + L_{ut}^f \leq W_t^L, \tag{A8}$$

$$H_t^f \equiv \sum_i H_{oit}^f + H_{ut}^f \tag{A9}$$

$$H_t^h \equiv \sum_i H_{oit}^h + H_{ut}^h \tag{A10}$$

$$H_t^m + H_t^f \leq W_t^H$$

(A11)

$h(L_{oit}^h) \in [0, L_{oit}^h]$, and (A12)

$$h(L_{ut}^h) \in [0, L_{ut}^h]. \tag{A13}$$

Given that the household's utility function is strictly increasing, preferences are locally non-satiated and so the constraints in equations (A4), (A8) are (A11) binding. The household allocates labor and land conditional on its expectations regarding its *ex post* optimal choices of c_o, c_u, and ℓ.

By Epstein's (1975) duality result, we can use the household's variable indirect utility function $V(\cdot)$, which is homogeneous of degree zero in prices and income, i.e., the measurement unit chosen to measure prices and income do not matter. Thus, dropping subscripts, we can set the price of the unobserved commodity p_u as numéraire, so that $p_i = p_{oi}/p_u$ and $Ey = E[Y^*/p_u]$. We also assume that the household is Arrow-Pratt income risk-averse, in the sense that $\frac{\partial^2 V}{\partial y^2} = V_{yy} < 0$.[3] Finally, note that in what follows, we assume away output and income volatility in order to focus solely on the effects of price volatility.

Using the household's (variable) indirect utility function, we can drop the subscripts and rewrite the household's maximization problem as

$$\max_{\{H_{oi}^h, L_{oi}^h, H_{oi}^f, L_{oi}^f, L_u^h, L_u^f, H^m, \ell\}} EV(\ell, p_i, y) \tag{A14}$$

[3] In a slight abuse of notation, we use subscripts not only to denote commodities but also the partial derivatives of the function $V(\cdot)$ in what follows.

subject to

$$Y = w[W^L - \ell - \sum_i L_{oi}^f - \sum_i L_{oi}^h - L_u^f - L_u^h] + r[W^H - \sum_i H_{oi}^f - \sum_i H_{oi}^h - H_u^f - H_u^h]$$

$$+ \sum_i p_i F_{oi}(L_{oi}, H_{oi}) + F_u(L_u, H_u) + I\,. \qquad \text{(A15)}$$

The first-order necessary conditions (FONCs) for this problem are then:

with respect to L_{oi}^h: $E\left\{V_y\left(p_i \dfrac{\partial F_{oi}}{\partial L_{oi}^h} - w\right)\right\} \le 0$ ($=0$ if $L_{oi}^h > 0$), (A16)

with respect to H_{oi}^h: $E\left\{V_y\left(p_i \dfrac{\partial F_{oi}}{\partial H_{oi}^h} - r\right)\right\} \le 0$ ($=0$ if $H_{oi}^h > 0$), (A17)

with respect to L_{oi}^f: $E\left\{V_y\left(p_i \dfrac{\partial F_{oi}}{\partial L_{oi}^f} - w\right)\right\} \le 0$ ($=0$ if $L_{oi}^f > 0$), (A18)

with respect to H_{oi}^f: $E\left\{V_y\left(p_i \dfrac{\partial F_{oi}}{\partial H_{oi}^f} - r\right)\right\} \le 0$ ($=0$ if $H_{oi}^f > 0$), and (A19)

with respect to ℓ: $E\{V_\ell - V_y w\} \le 0$ ($=0$ if $\ell > 0$). (A20)

Intuitively, diminishing marginal utility of wealth implies that V_y is correlated with the terms in parentheses in equations (A16) to (A19), meaning the household will fail to maximize expected profit. Equation (A20) means that the household will set its (expected) marginal utility of leisure equal to the marginal cost of leisure. This set of

FONCs is similar to what is usually derived from the basic agricultural household model (Singh et al., 1986; Bardhan and Udry, 1999).

B. Deriving the Matrix of Price Risk Aversion Coefficients

Recall that by Roy's Identity, i.e., $M_i = \dfrac{\partial V / \partial p_i}{\partial V / \partial y}$,[4] we have that

$$V_y = \frac{V_{p_i}}{M_i} = \frac{V_{p_j}}{M_j}, \tag{B1}$$

where M_j is the marketable surplus of commodity *j*. Additionally,

$$V_{yp_j} = \left(\frac{V_{p_i p_j}}{M_i} - \frac{V_{p_i}}{M_i^2} \frac{\partial M_i}{\partial p_j} \right) = \frac{1}{M_i} \left\{ V_{p_i p_j} - \frac{\partial M_i}{\partial p_j} V_y \right\}. \tag{B2}$$

We also have that

$$M_i = \frac{V_{p_i}}{V_y} \Leftrightarrow V_{p_i} = M_i V_y, \tag{B3}$$

which implies that

$$V_{p_i p_j} = M_i V_{yp_j} + V_y \frac{\partial M_i}{\partial p_j}, \tag{B4}$$

[4] One can apply Roy's identity to the marketable surplus equation given that it is both additive and convex. See Barrett (1996) and Finkelshtain and Chalfant (1991).

which, in turn, implies that

$$V_{p_i y} = M_i V_{yy} + V_y \frac{\partial M_i}{\partial y} = V_{y p_i}, \tag{B5}$$

where the last equation is the result of applying Young's theorem on the symmetry of second derivatives, which requires that (i) $V(\cdot)$ be a differentiable function over (p, y); and (ii) its cross-partials exist and be continuous at all points on some open set.

Replacing V_{yp_i} by equation (B5) in equation (B4) yields

$$V_{p_i p_j} = M_i \left\{ M_j V_{yy} + V_y \frac{\partial M_j}{\partial y} \right\} + V_y \frac{\partial M_i}{\partial p_j}. \tag{B6}$$

Then, we have that

$$V_{p_i p_j} = M_i M_j V_{yy} + M_i V_y \frac{\partial M_j}{\partial y} + V_y \frac{\partial M_i}{\partial p_j}. \tag{B7}$$

Multiplying the first term by $V_y y / V_y y$ yields

$$V_{p_i p_j} = -\frac{M_i M_j R V_y}{y} + M_i V_y \frac{\partial M_j}{\partial y} + V_y \frac{\partial M_i}{\partial p_j}, \tag{B8}$$

where R is the household's Arrow-Pratt coefficient of relative risk aversion. Multiplying the second term by $M_j y / M_j y$ and the third term by $M_i p_j / M_i p_j$ yields

$$V_{p_i p_j} = -\frac{M_i M_j R V_y}{y} + M_i V_y \eta_j \frac{M_j}{y} + V_y \varepsilon_{ij} \frac{M_i}{p_j}, \tag{B9}$$

where η_j is the income-elasticity of the marketable surplus of commodity j and ε_{ij} is the elasticity of commodity i with respect to the price of commodity j. Equation (B9) is thus equivalent to

$$V_{p_i p_j} = M_i V_y \left[-\frac{M_j R}{y} + \eta_j \frac{M_j}{y} + \varepsilon_{ij} \frac{1}{p_j} \right]. \tag{B10}$$

Multiplying the first two terms in the bracketed expression by p_j / p_j yields

$$V_{p_i p_j} = \frac{M_i V_y}{p_j} \left[-R\beta_j + \eta_j \beta_j + \varepsilon_{ij} \right], \tag{B11}$$

where β_j is the budget share of commodity *j*. When simplified, equation (B11) becomes such that

$$V_{p_i p_j} = \frac{M_i V_y}{p_j}\left[\beta_j(\eta_j - R) + \varepsilon_{ij}\right]. \tag{B12}$$

Consequently, if $M_i = 0$, the household is indifferent to volatility in the price of good *i* (i.e., the variance in the price of good *i*) and to covolatility in the prices of goods *i* and *j* (i.e., the covariance between the prices of good *i* and *j*) since its autarky from the market leaves it unaffected at the margin by price volatility.

Applying Young’s theorem once again, we obtain the following equation:

$$V_{p_i p_j} = \frac{M_i V_y}{p_j}\left[\beta_j(\eta_j - R) + \varepsilon_{ij}\right] = \frac{M_j V_y}{p_i}\left[\beta_i(\eta_i - R) + \varepsilon_{ji}\right] = V_{p_j p_i}. \tag{B13}$$

$$A_{ij} = \frac{V_{p_i p_j}}{V_y} = \frac{M_i}{p_j}\left[\beta_j(\eta_j - R) + \varepsilon_{ij}\right]$$

C. Proof of Proposition 1

Symmetry of the Slutsky matrix implies that

$$\frac{\partial M_i}{\partial p_j}+\frac{\partial M_i}{\partial y}M_j=\frac{\partial M_j}{\partial p_i}+\frac{\partial M_j}{\partial y}M_i\,. \tag{C1}$$

By Roy’s Identity, the above statement can be rewritten as

$$\frac{\partial}{\partial p_j}\left(-\frac{V_{p_i}}{V_y}\right)+\frac{\partial}{\partial y}\left(-\frac{V_{p_i}}{V_y}\right)\cdot\left[-\frac{V_{p_j}}{V_y}\right]=\frac{\partial}{\partial p_i}\left(-\frac{V_{p_j}}{V_y}\right)+\frac{\partial}{\partial y}\left(-\frac{V_{p_j}}{V_y}\right)\cdot\left[-\frac{V_{p_i}}{V_y}\right], \tag{C2}$$

which, once the second-order partials are written explicitly, is equivalent to

$$-\left(\frac{V_{p_i p_j}V_y-V_{yp_j}V_{p_i}}{V_y^2}\right)+\left(\frac{V_{p_i y}V_y-V_{yy}V_{p_i}}{V_y^2}\right)\cdot\left[\frac{V_{p_j}}{V_y}\right]=$$
$$-\left(\frac{V_{p_j p_i}V_y-V_{yp_i}V_{p_j}}{V_y^2}\right)+\left(\frac{V_{p_j y}V_y-V_{yy}V_{p_j}}{V_y^2}\right)\cdot\left[\frac{V_{p_i}}{V_y}\right]. \tag{C3}$$

This last equation can then be arranged to show that

$$\left(V_{p_i p_j}-V_{p_j p_i}\right)V_y=V_{yp_j}V_{p_i}-V_{p_j y}V_{p_i}-V_{yp_i}V_{p_j}+V_{p_i y}V_{p_j}\,. \tag{C4}$$

By Young’s Theorem, we know that $V_{p_i p_j} = V_{p_j p_i}$, that $V_{yp_i} V_{p_j} = V_{p_i y} V_{p_j}$, and that $V_{yp_j} = V_{p_j y}$, so both sides of the previous equation are identically equal to zero. In other words, symmetry of the Slutsky matrix implies and is implied by symmetry of the matrix A of price risk aversion coefficients.

D. Deriving Household WTP to Stabilize Prices

We model risky choice as a two period model in which decisions are made in the first period, and prices (and thus utility) are realized in the second period. We can then define the willingness to pay for eliminating all price risk implicitly as

$$E[V(E(p), y - WTP)] = E\big(V(p, y)\big) \qquad \text{(D1)}$$

where exogenous income y may be random but is uncorrelated with stochastic prices. We can then proceed as is common (see Arrow, 1971; Sandmo 1971) using Taylor series expansions to approximate (D1). Following the standard procedure in the literature (Arrow, 1971), we approximate the terms of the (D1) equation using a first order Taylor series expansion in directions of certainty (i.e., the elements of (p)) around the mean price, and using a second order Taylor series expansion around mean price and income in all dimensions involving risk (i.e., the elements of p, and for changes in y).[5] This results in:

$$\begin{aligned} &E\big[V(\mu_p, \mu_y) + V_y(\mu_p, \mu_y)(y - WTP - \mu_y)\big] \\ &\quad - E\Big[V(\mu_p, \mu_y) + V_p(\mu_p, \mu_y)(p - \mu_p) + V_y(\mu_p, \mu_y)(y - \mu_y) \\ &\quad + \frac{1}{2}(p - \mu_p)' V_{pp}(\mu_p, \mu_y)(p - \mu_p) + \frac{1}{2}(y - \mu_y)^2 V_{yy}(\mu_p, \mu_y) \\ &\quad + \frac{1}{2}(p - \mu_p)' V_{py}(\mu_p, \mu_y)(y - \mu_y) + \frac{1}{2}(y - \mu_y)' V_{yp}(\mu_p, \mu_y)(p - \mu_p)\Big] \\ &\quad = 0 \end{aligned}$$

[5] This is typically justified by claiming that the measure of WTP is small (e.g., Wright and Williams 1988). However in this case WTP may make up a substantial portion of income. The true requirement for this to be a relatively accurate measure is for WTP to be small relative to the variance of wealth. Given reasonable levels of relative risk aversion assumed in this manuscript, this is virtually assured (Just, 2011).

(D2)

Dividing both sides by $V_y(\mu_p, \mu_y)$, and collecting terms we can rewrite this last equation as

$$WTP = -E\left[\frac{1}{2}(p-\mu_p)'\frac{V_{pp}(\mu_p,\mu_y)}{V_y(\mu_p,\mu_y)}(p-\mu_p) + \frac{1}{2}(p-\mu_p)'\frac{V_{py}(\mu_p,\mu_y)}{V_y(\mu_p,\mu_y)}(y-\mu_y) + \frac{1}{2}(y-\mu_y)'\frac{V_{yp}(\mu_p,\mu_y)}{V_y(\mu_p,\mu_y)}(p-\mu_p)\right]$$

(D3)

By taking the expectations, equation (D4) can be written more simply as

$$WTP = -\tfrac{1}{2}\left[\sum_{j=1}^{k}\sum_{i=1}^{k}\sigma_{ij}\frac{V_{p_j p_i}}{V_y} + 2\sum_{i=1}^{k}\frac{V_{yp_i}}{V_y}\sigma_{yi}\right] \qquad \text{(D4)}$$

If we assume that exogenous income (which is likely to be locally determined) is uncorrelated with prices (which are likely to be globally determined), then this simplifies to [6]

$$WTP = -\tfrac{1}{2}\left[\sum_{j=1}^{k}\sum_{i=1}^{k}\sigma_{ij}\frac{V_{p_j p_i}}{V_y}\right] \qquad \text{(D5)}$$

If we are only stabilizing one price, then willingness to pay is defined by:

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[6] If instead we take a second order approximation of all terms of (D1), we find the more accurate measure $WTP = -\frac{y}{R}\left[1 - \sqrt{1 - \frac{R}{y}\left[\sum_{j=1}^{k}\sum_{i=1}^{k}\sigma_{ij}\frac{V_{p_j p_i}}{V_y} + 2\sum_{i=1}^{k}\frac{V_{yp_i}}{V_y}\sigma_{yi}\right]}\right]$ The measure we employ is a monotonic transform of this more accurate measure, with both measures taking on the same sign except in the case where income is zero, in which case the more accurate measure is not defined. Due to the substantive number of households with zero income in the ERHS data, we use the less accurate measure, which may positively bias the willingness to pay estimates but without changing the signs of the estimates. In datasets where this numerical problem would not be encountered, the more accurate estimate is clearly preferable.

$$E[V(E(p_i), p_{\sim i}, y - WTP)] = E\big(V(p_i, p_{\sim i}, y)\big) \qquad \text{(D6)}$$

where p_{-i} is the random vector of all prices except for that of commodity i. Equation (D6) can now be approximated as before

$$E\begin{bmatrix} V(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y) + V_y(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)(y - WTP - \mu_y) \\ +V_{p_{\sim i}}(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)(p_{\sim i} - \mu_{p_{\sim i}}) + \frac{1}{2}(p_{\sim i} - \mu_{p_{\sim i}})' V_{p_{\sim i}p_{\sim i}}(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)(p_{\sim i} - \mu_{p_{\sim i}}) \\ +(p_{\sim i} - \mu_{p_{\sim i}})' V_{p_{\sim i}y}(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)(y - \mu_y) \end{bmatrix} -$$

$$E\begin{bmatrix} V(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y) + V_{p_{\sim i}}(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)(p_{\sim i} - \mu_{p_{\sim i}}) + V_{p_i}(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)(p_i - \mu_{p_i}) \\ +V_y(\mu_p, \mu_y)(y - \mu_y) + \frac{1}{2}(p_{\sim i} - \mu_{p_{\sim i}})' V_{p_{\sim i}p_{\sim i}}(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)(p_{\sim i} - \mu_{p_{\sim i}}) \\ +\frac{1}{2}(p_i - \mu_{p_i})' V_{p_ip_i}(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)(p_i - \mu_{p_i}) \\ +\frac{1}{2}(p_i - \mu_{p_i})' V_{p_ip_{\sim i}}(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)(p_{\sim i} - \mu_{p_{\sim i}}) \\ +\frac{1}{2}(p_{\sim i} - \mu_{p_{\sim i}})' V_{p_{\sim i}p_i}(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)(p_i - \mu_{p_i}) \\ +(p_{\sim i} - \mu_{p_{\sim i}})' V_{p_{\sim i}y}(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)(y - \mu_y) \\ +(p_i - \mu_{p_i})' V_{p_iy}(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)(y - \mu_y) \end{bmatrix} = 0$$

(D7)

Dividing both sides of (D7) by $V_y(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)$ and collecting terms results in

$$WTP = -E\Bigg[\frac{1}{2}(p_i - \mu_{p_i})' \frac{V_{p_ip_{\sim i}}(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)}{V_y(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)}(p_{\sim i} - \mu_{p_{\sim i}})$$

$$+\frac{1}{2}(p_{\sim i} - \mu_{p_{\sim i}})' \frac{V_{p_{\sim i}p_i}(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)}{V_y(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)}(p_i - \mu_{p_i})$$

$$+\frac{1}{2}(p_i - \mu_{p_i})' \frac{V_{p_ip_i}(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)}{V_y(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)}(p_i - \mu_{p_i})$$

$$+(p_i - \mu_{p_i})' \frac{V_{p_iy}(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)}{V_y(\mu_{p_i}, \mu_{p_{\sim i}}, \mu_y)}(y - \mu_y)\Bigg]$$

Carrying out the expectation leads to[7]

$$WTP = -\left[\frac{1}{2}\sigma_{ii}\frac{V_{p_i p_i}}{V_y} + \sum_{j\neq i}\sigma_{ji}\frac{V_{p_j p_i}}{V_y} + \sigma_{yi}\frac{V_{p_i y}}{V_y}\right]$$

If income is uncorrelated with prices, then WTP simplifies to

$$WTP = -\frac{1}{2}\sigma_{ii}\frac{V_{p_i p_i}}{V_y} - \sum_{j\neq i}\sigma_{ji}\frac{V_{p_j p_i}}{V_y}$$

[7] As in footnote 6 above, a more accurate measure of willingness to pay is possible by employing $WTP = -\frac{y}{R}\left(1 - \sqrt{1 - \frac{R}{y}\left(\frac{R}{y}\sigma_y^2 + \left[\frac{1}{2}\sigma_{ii}\frac{V_{p_i p_i}}{V_y} + \sum_{j\neq i}\sigma_{ji}\frac{V_{p_j p_i}}{V_y} + \sigma_{yi}\frac{V_{p_i y}}{V_y}\right]\right)}\right)$. However, we have chosen to use the less accurate approximation given the relative frequency of households with zero income, and the numerical problems that result.

E. Identification Strategy

One complicating factor is that income is endogenous in the theoretical model (i.e., the agricultural household model) that underlies our empirical analysis – see, for example, equations A4 and A5 in appendix A. Both equations combined show that the Beckerian full-income constraint of the household.

In the empirical framework, recall that statistical endogeneity problems (which lead to coefficients being biased, and thus not identified) can arise from:

1. Unobserved heterogeneity,
2. Measurement error, and
3. Reverse causality (or simultaneity).

We now explicitly discuss how each of those sources of endogeneity could affect our estimation results, and explain how our identification strategy alleviates concerns about any resulting bias.

It is important to bear in mind that this problem is intrinsically not amenable to conventional methods for eliminating endogeneity due to the first or third causes. Although some researchers have successfully randomized price levels (i.e., treating them as fixed, not stochastic, variables) of one commodity by offering randomly assigned vouchers (e.g., for rice to rural Chinese households by Jensen and Miller, 2008), joint randomization of (or, more generally, instrumentation for) income and multiple commodities’ price distributions is clearly not feasible in this or any other context. So perfectly ‘clean’ identification is likely unattainable for this problem. The best that can be done is careful attention to and forthright declaration of these issues. In what follows, we argue that our identification strategy – which relies on longitudinal data, household fixed effects, and location-time fixed effects – is the best one can do, at least with the ERHS data and perhaps with any existing household data set. This is far too important an economic policy question to ignore out of concern for statistical perfection that is intrinsically unattainable in general equilibrium problems, which includes those associated with nonseparable agricultural household models of the sort we employ.

Unobserved Heterogeneity

The unit of observation in this article is the household. In this context, unobserved heterogeneity can arise for a multitude of reasons, all having to do with the fact that households differ from one another in systematic ways that are correlated with the regressors in our core specification (i.e., commodity prices and household income).

The *commodity prices* we use are village-level prices, and so they vary within household over time and among households in different villages within a district in a given time period, given district-round fixed effects. As such, even if they are correlated with the unobserved heterogeneity among households (say, because the households in a village may have a stronger preference for a given commodity, which drives up the price of that commodity in the village relative to other villages), any time invariant component of this (e.g., related to preferences) should be controlled for by household fixed effects. Moreover, the spatial price analysis literature on food markets in Ethiopia shows that prices transmit quite well and quickly (e.g., Dercon 1995, Negassa and Myers 2007), so the likelihood is very low of unobserved, time-varying heterogeneity that is not already captured by the district-round dummies. Granted, it is possible that commodity prices are correlated with the unobserved heterogeneity between households – say, if several of the households in the data were to experience the same change in preferences about a given commodity, which would change the price of that commodity – but this seems highly unlikely.

Our measure of *income* is household-specific. Its coefficient is identified by the within-household variation in income over time as well as by the between-household-within-district variation in income at a given time. Income is determined by the household’s crop sales, its revenue from wages, its revenue from land (and other input) leases, and the transfers it receives from various sources. Income from crop sales ($P \cdot Q$) is jointly determined by the prices P the household receives for its crops, which are explicitly controlled for (and are uncorrelated with unobserved household heterogeneity given that they are village-level prices), and by the quantity Q it produces. For the households whose production decision is separable from their consumption decision (i.e., for the households for whom the Separation Property holds; see Singh et al., 1986), the quantity Q produced by the household is determined by input and output prices and by technology. Input prices are controlled for here by our use of district-round fixed effects. The technology employed by the households in our data is everywhere the same – even the largest landholders in the data produce using very primitive technology, with no mechanization. For the households whose production decision is not separable from their consumption decision (i.e., for the households for whom the Separation Property does not hold), Q is also determined by the preferences and endowments of the households. Household fixed effects control for time invariant household preferences and for the average household endowments of land, labor, etc. over time. For other sources of income, such as wage receipts, revenue from land (and other input leases), and transfers from various sources, household fixed effects control for the within-household average of those variables. In the case of wage receipts and revenue from input leases, part of those variables is determined by input prices, which are controlled for by district-round fixed effects.

What remains unaccounted for, then, are systematic departures from the household average of each income category. But those systematic departures are largely driven by unanticipated shocks (e.g., a member of the household gets sick and the household cannot produce as much and does not receive as much as usual in terms of wage receipts, market demand fluctuates; there are weather shocks which affect production, and so on). Thus, while there may be some residual correlation between household income and the error term of our core equation, that surely represents a very small part of the variation in income as we have controlled for the major likely sources from which such correlation can arise. Moreover, as previously discussed, instrumentation for income and price distributions jointly is infeasible, so this seems the best that can be done with any feasible household data set.

Measurement Error

It is unlikely that commodity prices are measured with error. But we can think of no reason why there would be any systematic pattern to measurement error in prices, so this will merely lead to attenuation bias in the price elasticity estimates, which would bias price risk aversion parameter estimates toward zero.

Looking at the household income data, it appears likely to have been systematically under-reported in this context. This is evident when looking at the proportion of zero incomes in the data (i.e., roughly 25 percent of observations). The inclusion of household fixed effects takes care of this measurement error problem in so far as misreporting is systematic because of time invariant respondent characteristics (e.g., a propensity to lie about their income, forgetfulness, etc.). The survey protocol made sure to always resurvey the same respondents (i.e., the household head for the production module, his spouse for the consumption and health module, etc.). Though it is likely that respondents might not be as forgetful from time to time, or that they might not be as likely to lie about their income by the same proportion every single time, it is unclear why those departures from the average in terms of forgetfulness, propensity to lie, etc. would be correlated with the RHS variables in any systematic way. So the remaining problem is once again classical measurement error and attenuation bias in the income elasticity of marketable surplus coefficient, which would bias our price risk aversion estimates toward zero.

Reverse Causality

Because the commodity prices we use as our RHS variable are community-level observations, it is highly unlikely that any single household's marketable surplus of a given commodity affects the community-level price of that commodity. In other words, although some households produce or consume more than others, no household sets prices in these data, so reverse causality is not a problem for prices.

On household income, reverse causality is an issue if an increase in marketable surplus (i.e., the dependent variable) causes an increase in income. Indeed, income certainly is endogenous in the theoretical model given that one component is the value of crop sales. But that theoretical endogeneity does not automatically imply statistical endogeneity. We now turn to explaining why one should not worry too much about reverse causality between marketable surplus and income in these data.

In short, our reasoning is much the same as in the case of unobserved heterogeneity. The statistically independent components of crop sales income, given explicit controls for prices and household and district-round fixed effects, are deviations from the household intertemporal means of those variables that determine output quantity. These are likely driven by unanticipated shocks (e.g., a member of the household gets sick and the household cannot produce as much and does not receive as much as usual in terms of wage receipts; within-district weather shocks affect production; and so on) or are predetermined, with no evidence of residual autocorrelation in errors (e.g., past periods’ marketable surplus enables input acquisition that expands subsequent period output and thus income). Again, while it is undeniably true that there will be some amount of correlation between household income and the error term of our core equation, we believe that represents a very small part of the variation in income and that we have controlled for many of the possible sources from which such correlation can arise.

Endogeneity of Income, Redux

Given the inevitable residual correlation between income and the error term in our core equation, our coefficient estimates are certainly not “causal” impact estimates. But this is a context where joint randomization of incomes and prices is simply not possible and credible instruments are not available, as would be typical of virtually any such setting. As such, our design is the best available design to answer the question we set out to answer. There is not much that can be done to try to eliminate whatever statistical endogeneity remains after the various efforts we have made to ameliorate such concerns.

Some commentators have suggested that we should use weather as an IV for income. Weather unfortunately cannot be used as an IV given (i) the small number of villages (which would lead to the coefficient on income being estimated only off of too small a number of observations) and (ii) the inclusion of district-round dummies, which already control for what a weather variable would do in this context. Moreover, any weather data would be either from meteorological stations at some (non-negligible) distance from most of these very small rural villages, or based on a very rough imputation over space among remotely sensed and terrestrial meteorological station data. Either way, there would be a considerable amount of measurement error in the temperature or rainfall data for a small number of villages, so this is clearly not a solution. Finally, a recent working paper by Sarsons (2011) seriously questions the use of weather or rainfall as an instrumental variable.

Our core contribution lies in deriving the analytical expression for multivariate price risk aversion and in laying out an estimation strategy and generating plausible – but certainly not definitive – empirical estimates that can usefully inform policy dialogue. These contributions are not at all compromised by the likely modest endogeneity of the income regressor; the empirical contribution of our article is merely an illustration of what is feasible. Recalling the statistician George Box's famous caution that "all models are wrong, but some are useful," we submit that it is very difficult to believe that the full range of parameters necessary to estimate price risk aversion coefficients in a multivariate setting could be estimated with clean identification with any data set. We do believe by any means that ours is the final word on this topic; we hope others will employ this (or improved) methods with other data to provide a broader range of estimates to inform policy discussion. All empirical results need to be treated with healthy skepticism; we go to considerable lengths to make that clear to readers. But given the high-level policy importance of the topic, it is incumbent on the profession to get this issue back into discussion after a long period of intellectual exile.

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